## SECTION 9

### OVERHEAD TRANSMISSION LINES AND UNDERGROUND CABLES

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*Original author of this section was John S. Wade, Jr., Ph.D., The Pennsylvania State University.*

9.1
INTRODUCTION

Overhead transmission lines are composed of aluminium conductors that, even in modest capacities, are stranded in spiral fashion for flexibility. These are primarily classified as:

- AAC: all-aluminium conductors
- AAAC: all-aluminium alloy conductors
- ACSR: aluminium conductors, steel reinforced
- ACAR: aluminium conductors, alloy reinforced

Aluminium conductors are compared in conductivity with the International Annealed Copper Standard (IACS) in Table 9.1. The table lists the percent conductivity, as well as the temperature coefficient of resistance, \( \alpha \), expressed per °C above 20°C.

CONDUCTOR RESISTANCE

Calculate the resistance of 1000 ft (304.8 m) of solid round aluminum conductor, type EC-H19 (AWG No.1) at 20 and 50°C. The diameter = 0.2893 in. (7.35 mm).

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<th>Material</th>
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Calculation Procedure

1. Calculate Resistance at 20°C

Use \( R = \frac{pl}{A} \), where \( R \) is the resistance in ohms, \( p \) is the resistivity in ohm-circular mils per foot (\( \Omega \cdot \text{cmil/ft} \)), \( l \) is the length of conductor in feet, and \( A \) is the area in circular mils (cmil), which is equal to the square of the conductor diameter given in mils. From Table 9.1, the percentage of conductivity for EC-H19 is 61 percent that of copper. For IACS copper, \( p = 10.4 \Omega \cdot \text{cmil/ft} \). Therefore, for aluminum, the resistivity is \( 10.4/0.61 \Omega \cdot \text{cmil/ft} \). The resistance of the conductor at 20°C is \( R = \frac{(17.05)(1000)}{289.32} = 0.204 \Omega \).

2. Calculate Resistance at 50°C

Use \( R_T = R_{20°} [1 + \alpha(T - 20°)] \), where \( R_T \) is the resistance at the new temperature, \( R_{20°} \) is the resistance at 20°C, and \( \alpha \) is the temperature coefficient of resistance. From Table 9.1, \( \alpha = 0.00403 \). Hence, \( R_{50°} = 0.204[1 + 0.00403(50 - 20)] = 0.229 \Omega \).

INDUCTANCE OF SINGLE TRANSMISSION LINE

Using flux linkages, determine the self-inductance, \( L \), of a single transmission line.

Calculation Procedure

1. Select Appropriate Equation

Use \( L = \frac{\lambda}{i} \), where \( L \) is the self-inductance in H/m, \( \lambda \) is the magnetic flux linkage in Wb/turns/m, and \( i \) is the current in amperes.

2. Consider Magnetic Flux Linkage

Assume a long, isolated, round conductor with uniform current density. The existing flux linkages include those internal to the conductor partially linking the current and those external to the conductor, which links all of the current. These will be calculated separately and then summed, yielding total inductance, \( L_T = L_{\text{int}} + L_{\text{ext}} \).

3. Apply Ampere’s Law

The magnitude of the magnetic flux density, \( B \), in Wb/m² produced by a long current filament is \( B = \frac{\mu i}{2\pi r} \), where \( \mu = \) the permeability of the flux medium (4\( \pi \times 10^{-7} \text{ H/m} \) for free space and nonferrous material) and \( r = \) radius to \( B \) from the current center in meters. The direction of \( B \) is tangential to encirclements of the enclosed current, and clockwise if positive current is directed into this page (right-hand rule). The differential flux linkages per meter external to a conductor of radius \( a \) meters are \( d\lambda = \frac{\mu i}{2\pi r} dr \) Wb-turns/m and \( r > a \).

4. Consider Flux Inside Conductor

The calculation of differential flux linkages is complicated by \( B \), which is a function of only that part of the current residing inside the circle passing through the measuring point of \( B \). The complication is compounded by the reduction in current directly affecting \( d\lambda \). Thus, if \( B = (\mu i/2\pi r)(r^2/\pi a^2) \) Wb/m², then \( d\lambda = (\mu i/2\pi r)(r^2/\pi a^2) \) = \( (\mu i/2\pi)(r^2/a^2) \) Wb-turns/m for \( r < a \).
5. **Calculate the Internal Inductance**

Integrating the expression in (4) yields: \( \lambda = (\mu i/2\pi)(r'/a) \), from which:

\[
L_{\text{int}} = (10^{-7}/2) \text{ H/m.}
\]

**Related Calculations.** Calculation of the inductance due to the flux external to a long isolated conductor yields an infinite value, since \( r \) varies from \( a \) to infinity, but then such an isolated conductor is not possible.

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**INDUCTANCE OF TWO-WIRE TRANSMISSION LINE**

Consider a transmission line consisting of two straight, round conductors (radius \( a \) meters), uniformly spaced \( D \) meters apart, where \( D \gg a \) (Fig. 9.1). Calculate the inductance of the line.

**Calculation Procedure**

1. **Consider Flux Linkages**

   It is realistic to assume uniform and equal but opposite current density in each conductor. The oppositely directed currents, therefore, produce a net total flux linkage of zero because the net current in any cross section of both conductors is zero. This is true for any multiconductor system whose cross-sectional currents add to zero (for example, in a balanced three-phase system).

![Diagram of a two-wire transmission line](image)

**FIGURE 9.1** A two-wire transmission line.

2. **Calculate Inductance of One Conductor**

   Use \( \lambda = (\mu i/2\pi)(r'/a) \ln(D/a) \) Wb \cdot \text{turns/m. Because } L = \lambda/i, L = (2 \times 10^{-7}) \left( \frac{1}{a} + \ln(D/a) \right) \text{ H/m. Inductance } L \text{ may be expressed in more compact form by } L = (2 \times 10^{-7}) \ln(D/r'), \text{ where } r' = a \exp(-\frac{D}{a}) \text{ is the geometric mean radius, GMR. The value of } r' \text{ is } 0.788a.

3. **Calculate Total Inductance, } L_T$$

\[
L_T = 2L = (4 \times 10^{-7})\ln(D/r') \text{ H/m. In more conventional units, } L_T = 1.482 \times \log(D/r') \text{ mH/mi.}
\]

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**INDUCTIVE REACTANCE OF TWO-WIRE TRANSMISSION LINE**

Calculate the inductive reactance of 10 mi (16.1 km) of a two-conductor transmission line (Fig. 9.1), where \( D = 8 \text{ ft (2.44 m)} \) and \( a = 0.1 \text{ in. (2.54 mm)} \) at a frequency of 60 Hz (377 rad/s).
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Calculation Procedure

1. **Calculate the Geometric Mean Radius**
   The GMR is \( r' = (0.7788)(2.54 \times 10^{-3}) = 0.001978 \text{ m} \).

2. **Calculate \( L_T \)**
   \[ L_T = (4 \times 10^{-7}) \ln(D/r') = (4 \times 10^{-7}) \ln(2.44/0.001978) = 28.5 \times 10^{-7} \text{ H/m}. \]

3. **Calculate Inductive Reactance \( X_L \)**
   \[ X_L = (377)(28.5 \times 10^{-7} \text{ H/m})(16.1 \times 10^{3} \text{ m}) = 17.3 \text{ \Omega}. \]

**Related Calculations.** A larger conductor and/or smaller conductor spacing reduces the inductive reactance.

**INDUCTANCE OF STRANDED-CONDUCTOR TRANSMISSION LINE**

Determine the inductance of a transmission line having six identical round conductors (Fig. 9.2) arranged so that the currents on each side occupy the conductors equally with uniform current density.

Calculation Procedure

1. **Calculate Flux of One Conductor**
   The flux linkages of conductor 1, which carries one-third of the current, can be deduced from the method established for the two-conductor line by:
   \[ \lambda_1 = (\mu/2\pi) \left( \frac{i}{3} \right) \left[ \frac{1}{\ln(D_{11}/a)} + \ln(D_{12}/a) + \ln(D_{13}/a) - \ln(D_{21}/a) - \ln(D_{22}/a) \right], \]
   where \( D_{ij} \) is the distance between conductors \( i \) and \( j \) and so on. When terms are collected, the equation becomes:
   \[ \lambda_1 = (\mu/2\pi) \left[ \ln[(D_{11}D_{12}D_{13})^{1/3}]/(r'D_{12}D_{13})^{1/3} \right] \text{ Wb \cdot turns/m}. \]

2. **Calculate the Inductances**
   The inductances of conductors one, two, and three then become:
   \[ L_1 = \lambda_1/(i/3) = (3 \times 2 \times 10^{-7})[(\ln(D_{11}D_{12}D_{13})^{1/3})(r'D_{12}D_{13})^{1/3}] \text{ H/m}, \]
   \[ L_2 = \lambda_2/(i/3) = (3 \times 2 \times 10^{-7}) \ln[(D_{21}D_{22}D_{23})^{1/3}]/(r'D_{23}D_{13})^{1/3} \text{ H/m}, \]
   and \[ L_3 = \lambda_3/(i/3) = (3 \times 2 \times 10^{-7}) \ln[D_{31}D_{32}/(r'D_{23}D_{13})^{1/3}] \text{ H/m}. \]

**FIGURE 9.2** Stranded-conductor transmission line.
The inductance of the unprimed side (Fig. 9.2) of the line is one-third the average value because the inductances are in parallel. Then 
\[ L_{\text{avg}} = \frac{L_1 + L_2 + L_3}{3} \text{ H/m} \]
and 
\[ L = \left( L_1 + L_2 + L_3 \right)/9 \text{ H/m}. \]

3. Determine the Total Inductance, \( L_T \)
Combining the sets of equations in Step 2, we obtain:
\[ L_T = \frac{2 \times 10^{-7}}{n} \left[ \ln(D_{12}D_{13})D_{21} \right] \text{ H/m}. \]

Related Calculations. To find the inductance of the primed side of the line in Fig. 9.2, follow the same procedure as above. Again, summing produces the total inductance.

The root of the product in the denominator for the expression of \( L_T \) is a GMR. (The root of the product in the numerator is called the geometric mean distance, GMD.) Although tabulated values are usually available, one may calculate GMR:
\[ \text{GMR} = \left( r \sum D_{ij}^2 \right)^{1/n}, \]
where \( 1/n \) is the reciprocal of the number of strands.

INDUCTANCE OF THREE-PHASE TRANSMISSION LINES

Determine the inductance per phase for a three-phase transmission line consisting of single conductors arranged unsymmetrically (Fig. 9.3).

Calculation Procedure

1. Use Flux-Linkage Method
   The principle of obtaining inductance per phase by using the flux linkages of one conductor is utilized once again. If the line is unsymmetrical and it remains untransposed, the inductance for each phase will not be equal (transposition of a line occurs when phases \( a, b, \) and \( c \) swap positions periodically). Transposing a transmission line will equalize the inductance per phase. Inductance, however, varies only slightly when untransposed, and it is common practice, in hand calculations, to assume transposition as is done in what follows.

   Assume phase \( a \) shifts from position 1 to 2, and then to 3; phases \( b \) and \( c \) also move in the rotation cycle. The average flux linkages for phase \( a \) are then given by:
   \[ \lambda_a = \left( 2 \times 10^{-7} \right)/3 \left( \sum I_a, I_b, I_c \right) \left( D_{12} + D_{13} + D_{23} \right)/3 \text{ Wb \cdot turns/m}. \]

   FIGURE 9.3 A three-phase transmission line.
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2. Calculate L
Using \( r' \) and dividing by \( I_a \), we find the inductance per phase is

\[
L_\phi = \frac{2 \times 10^{-7}\ln(D_{12}D_{13}D_{23})^{1/3}/r'}{H/m}.
\]

**Related Calculations.** If the conductor for each phase is concentrically stranded, the distance between conductors remains the same, but \( r' \) is replaced by a tabulated GMR. The inductance in conventional form is given by:

\[
L_\phi = 0.741 \log\left[\frac{D_{12}D_{13}D_{23}}{GMR}\right] \text{ mH/mi; } D \text{ and GMR are given in feet.}
\]

**PER-PHASE INDUCTIVE REACTANCE**
Calculate the per-phase inductive reactance per mile (1600 m) for a three-phase line at 377 rad/s. The conductors are aluminum conductors, steel-reinforced (ACSR) Redwing (Table 9.2) arranged in a plane as shown in Fig. 9.4.

**Calculation Procedure**
1. Calculate \( L_\phi \)

From Table 9.2, GMR = 0.0373 ft (0.01 m). Substituting in the equation for per-phase inductance, we find

\[
L_\phi = \frac{2 \times 10^{-7}\ln(3.6 \times 7.2 \times 3.6)^{1/3}/0.01 = 12.2 \times 10^{-7} \text{ H/m}}{0.74 \text{ mH/mi.}}
\]

2. Calculate Inductive Reactance, \( X_L \)

\[
X_L = 377 \times 12.2 \times 10^{-7} \text{ H/m} \times 1600 \text{ m} = 0.74 \text{ }\Omega/\text{mi.}
\]

**INDUCTANCE OF SIX-CONDUCTOR LINE**
Calculate the per-phase inductance of the transmission line of Fig. 9.5 where the conductors are arranged in a double-circuit configuration.

**Calculation Procedure**
1. Use Suitable Expression for \( L_\phi \)

Use

\[
L_\phi = \frac{2 \times 10^{-7}\ln(GMD/GMR)}{H/m, \text{ where GMR is the GMR of a conductor.}}
\]

2. Calculate GMD

The GMD includes the distances between all the phase combinations. However, the expression for GMD can be reduced to one-half the distances that are represented in the original expression and the root becomes \( \sqrt[6]{g} \) rather than \( \sqrt[6]{g} \). Thus,

\[
GMD = (D_{a1b}D_{a1c}D_{a2b}D_{a2c}D_{b1c}D_{b1c})^{1/6} \text{ m.}
\]
TABLE 9.2  Aluminum Conductors, Steel Reinforced (ACSR)

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<tr>
<th>Code word</th>
<th>Size, Mcmil</th>
<th>Stranding aluminum/steel</th>
<th>Outside diameter, in</th>
<th>Resistance DC, Ω/1000 ft at 20°C</th>
<th>Resistance AC, 60 Hz, Ω/mi at 25°C</th>
<th>GMR, ft</th>
<th>Phase-to-neutral, 60 Hz, reactance at 1-ft spacing</th>
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<td>0.1304</td>
<td>0.0349</td>
<td>0.407 0.0932</td>
</tr>
<tr>
<td>Starling</td>
<td>715.5</td>
<td>26/7</td>
<td>1.081</td>
<td>0.0238</td>
<td>0.1294</td>
<td>0.0355</td>
<td>0.405 0.0948</td>
</tr>
<tr>
<td>Redwing</td>
<td>715.5</td>
<td>30/19</td>
<td>1.092</td>
<td>0.0237</td>
<td>0.1287</td>
<td>0.0373</td>
<td>0.399 0.0920</td>
</tr>
</tbody>
</table>
3. Calculate GMR

Use \( GMR = (GMR^c D_{al1} D_{bk2} D_{cl2})^{1/6} \) m.

INDUCTIVE REACTANCE OF SIX-CONDUCTOR LINE

Calculate the per-phase inductive reactance of a six-conductor, three-phase line at 377 rad/s consisting of Teal ACSR conductors (Fig. 9.5). Distance \( D_{al2} = 4.8 \) m, \( H_1 = H_2 = 2.4 \) m, and \( D_{bk2} = 5.4 \) m.

Calculation Procedure

1. Determine GMD

The necessary dimensions for calculating GMD are: 
\[
D_{al1} = D_{bk1} = (2.4^2 + 5.1^2)^{1/2} = 5.64 \text{ m},
D_{al1} = D_{bk1} = (2.4^2 + 0.3^2)^{1/2} = 2.42 \text{ m},
D_{cl1} = 4.8 \text{ m},
\]
and \( D_{cl2} = 4.8 \text{ m}. \) Hence, \( GMD = [(2.4^2)(5.64^2)(4.8^2)]^{1/6} = 4.03 \text{ m}. \)

2. Determine GMR

From Table 9.2, for Teal, \( GMR^c = 0.0341 \text{ ft} (0.01 \text{ m}). \) Then, 
\[
D_{al2} = D_{bk2} = (4.8^2 + 4.8^2)^{1/2} = 6.78 \text{ m},
D_{bk2} = 5.4 \text{ m},
\]
and \( GMR = [(0.01^3)(6.78^2)(5.4)]^{1/6} = 0.252 \text{ m}. \)

3. Calculate Inductive Reactance per Phase

\( \frac{X_L}{H} = 377(2 \times 10^{-7})[\ln(4.03/0.252)] = 0.209 \times 10^{-3} \text{ m}^{-1} (0.336 \text{ m}^{-1}). \)

INDUCTIVE REACTANCE OF BUNDLED TRANSMISSION LINE

Calculate the inductive reactance per phase at 377 rad/s for the bundled transmission line whose conductors are arranged in a plane shown in Fig. 9.6. Assume conductors are ACSR Crow.

Calculation Procedure

1. Determine GMD

Assume distances are between bundle centers and transposition of phases. Then, 
\( GMD = [(9^2)(18)]^{1/3} = 11.07 \text{ m}. \) From Table 9.2, \( GMR^c = 0.034 \text{ ft} (0.01 \text{ m}). \) The GMR

![Figure 9.6](image-url)
should include all conductor spacings from each other in the usual product form with, in this case, three values of $GMR_c$. Because of redundancy, $GMR = (0.01 \times 0.45^3)^{1/3} = 0.127 \text{ m}$.

2. Calculate Inductive Reactance per Phase

$$X_L = 377 \times 2 \times 10^{-7} \times \ln(11.07/0.127) = 0.337 \times 10^{-3} \Omega/m (0.544 \Omega/\text{mi}).$$

**Related Calculations.** For a two-conductor bundle, $GMR = (GMR_D)^{1/2}$ and for a four-conductor bundle, $GMR = (GMR_D^3)^{1/4}$. In each case, $D$ is the distance between adjacent conductors.

For a two-conductor bundle, $GMR = (GMR,D)^{1/2}$ and for a four-conductor bundle, $GMR = (GMR,D^3)^{1/4}$. In each case, $D$ is the distance between adjacent conductors.

For an $n$-conductor bundle as depicted in Fig. 9.7, $GMR = (n \cdot GMR, \cdot A^{-1})^{1/n}$, where $A$ is the radius of the bundle. Similarly, for an $n$-conductor bundle, $r_{equiv} = (n \cdot r \cdot A^{-1})^{1/n}$, where $r$ is the external radius of the conductor and $r_{equiv}$ is the equivalent external radius of the bundle (Dommel, 1992).

At voltage levels above 230 kV, the corona loss surrounding single conductors, even though they are expanded by nonconducting central cores, becomes excessive. Therefore, to reduce the concentration of electric-field intensity, which affects the level of ionization, the radius of a single conductor is artificially increased by arranging several smaller conductors, in what approximates a circular configuration. This idea is depicted in Fig. 9.7. Other arrangements that prove satisfactory, depending on the voltage level, are shown in Fig. 9.8.

A benefit that accrues from bundling conductors of one phase of a line is an increase in $GMR$. Also, the inductance per phase is reduced, as is corona ionization loss.

**INDUCTIVE REACTANCE DETERMINED BY USING TABLES**

Determine the inductive reactance per phase using data in Tables 9.2 and 9.3 for ACSR Redwing with the spacing given in Fig. 9.9.

**Calculation Procedure**

1. **Use Appropriate Tables**

The Aluminum Electrical Conductor Handbook provides tabulated data that reduce the amount of calculation necessary to find the inductive reactance for either a single- or three-phase line where circuits are neither paralleled nor bundled. To determine the reactance by this
<table>
<thead>
<tr>
<th>Separation of conductors</th>
<th>In Feet</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>1</td>
<td>0.0841</td>
<td>0.0097</td>
<td>0.0187</td>
<td>0.0271</td>
<td>0.0349</td>
<td>0.0423</td>
<td>0.0492</td>
<td>0.0558</td>
<td>0.0620</td>
<td>0.0679</td>
<td>0.0735</td>
<td>0.0789</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.1333</td>
<td>0.1366</td>
<td>0.1399</td>
<td>0.1430</td>
<td>0.1461</td>
<td>0.1491</td>
<td>0.1520</td>
<td>0.1549</td>
<td>0.1577</td>
<td>0.1604</td>
<td>0.1631</td>
<td>0.1657</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.1682</td>
<td>0.1707</td>
<td>0.1732</td>
<td>0.1756</td>
<td>0.1779</td>
<td>0.1802</td>
<td>0.1825</td>
<td>0.1847</td>
<td>0.1869</td>
<td>0.1891</td>
<td>0.1912</td>
<td>0.1933</td>
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<tr>
<td>4</td>
<td>0.1953</td>
<td>0.1973</td>
<td>0.1993</td>
<td>0.2012</td>
<td>0.2031</td>
<td>0.2050</td>
<td>0.2069</td>
<td>0.2087</td>
<td>0.2105</td>
<td>0.2123</td>
<td>0.2140</td>
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<td></td>
</tr>
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<td>5</td>
<td>0.2174</td>
<td>0.2191</td>
<td>0.2207</td>
<td>0.2224</td>
<td>0.2240</td>
<td>0.2256</td>
<td>0.2271</td>
<td>0.2287</td>
<td>0.2302</td>
<td>0.2317</td>
<td>0.2332</td>
<td>0.2347</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.2361</td>
<td>0.2376</td>
<td>0.2390</td>
<td>0.2404</td>
<td>0.2418</td>
<td>0.2431</td>
<td>0.2445</td>
<td>0.2458</td>
<td>0.2472</td>
<td>0.2485</td>
<td>0.2498</td>
<td>0.2511</td>
<td></td>
</tr>
</tbody>
</table>

*From formula at 60 Hz, $X_d = 0.2794 \log_{10} d$, $d =$ separation in feet.
method, it is convenient to express the inductive reactance by: \( X_L = 2\pi f(2 \times 10^{-7})\ln(1/GMR) + 2\pi f(2 \times 10^{-7})\ln \text{ GMD} \), or \( X_L = 2\pi f(0.7441 \times 10^{-3})\log(1/GMR) + 2\pi f(0.7441 \times 10^{-3})\log \text{ GMD} \). The first term in the latter expression is the reactance at 1-ft spacing, which is tabulated in Table 9.2 for 60 Hz. The second term is the spacing component of inductive reactance tabulated in Table 9.3 for a frequency of 60 Hz. Conversion from English to SI units, and vice versa, may be required.

2. Determine GMD

A distance of 3.6 m = 12 ft. Hence, GMD = \( [(12^2)(24)]^{1/3} = 15.1 \text{ ft (4.53 m)} \).

3. Determine \( X_L \)

From Tables 9.2 and 9.3, \( X_L = 0.399 + 0.329 = 0.728 \Omega/\text{mi} \). In SI units, \( X_L = (0.728 \Omega/\text{mi})(1 \text{ mi}/1.6 \text{ km}) = 0.45 \Omega/\text{km} \).

**EFFECT OF MUTUAL FLUX LINKAGE**

Referring to Fig. 9.10, find the voltage, in V/m, induced in a nearby two-conductor line by the adjacent three-phase transmission line carrying balanced currents having a magnitude of 50 A.

Calculation Procedure

1. Define Approach

Flux linkages within the 1.2-m-wide plane of the two-conductor line from each phase of the transmission line shall be summed. Then, from Faraday’s law, the derivative of this result will yield the answer.

2. Calculate Distances from Each Involved Conductor

\( D_{11} = 2.4 + 3 = 5.4 \text{ m}, \quad D_{32} = 1.2 + 2.4 + 3 = 6.6 \text{ m}, \quad D_{13} = 3 \text{ m}, \quad D_{21} = 3 + 1.2 = 4.2 \text{ m}, \quad D_{12} = (0.6^2 + 4.2^2)^{1/2} = 4.24 \text{ m}, \) and \( D_{23} = (0.6^2 + 5.4^2)^{1/2} = 5.43 \text{ m} \).

3. Calculate Flux Linkages

\( \lambda = \mu_i(\pi f)\ln(D_{32}/D_{11}) + \mu_i(\pi f)\ln(D_{13}/D_{21}) + \mu_i(\pi f)\ln(D_{23}/D_{12}). \) This equation is a function of time. Substituting values in the expression and combining terms, we find \( \lambda = \sqrt{2}(33.364 \sin(\omega t + 20.07) \sin(\omega t - 120^\circ) + 24.74 \sin(\omega t + 120^\circ)) \times 10^{-7} \text{ Wb-turns/m, where } \omega = 2\pi f \).

4. Apply Faraday’s Law

The voltage per unit length is \( V = d\lambda/dt = \sqrt{2}(33.364 \omega \cos(\omega t + 20.07) \cos(\omega t - 120^\circ) + 24.74\omega \cos(\omega t + 120^\circ)) \times 10^{-7} \text{ V/m} \).

5. Determine \( V \)

Transforming to the frequency domain and rms values, one obtains \( V = (0.424 + j0.143) \times 10^{-3} \text{ V/m} = (0.68 + j0.23) \text{ V/mi} \).
Consider a transmission line consisting of \( n \) straight conductors. For the sake of simplicity, let us depict only the conductors \( i \) and \( k \) with their respective images, and assume that the per-phase ac resistance has been obtained from Table 9.2. Determine the self and mutual impedances of such a line taking into account the ground returns.

**Calculation Procedure**

1. **Calculate the Complex Penetration Depth**

   Traditionally, ground-return corrections have been carried out by using series-based asymptotic approximation of Carson’s infinite integrals (Carson, 1926). However, truncation errors might be unacceptable for the impedances in cases with wide separation among conductors, frequency higher than power frequency, or low earth resistivity (Dommel, 1985).

   To represent current return through homogeneous ground, the ground can be replaced by an ideal plane placed below the ground surface at a distance equal to the complex penetration depth for plane waves (Fig. 9.11). Such an approach has been proposed in Dunbanton (1969), Gary (1976), and Deri et al. (1981) and produces results that match those obtained from Carson’s correction terms. The main advantage of this method is that it allows use of simple formulae for self and mutual impedances—formulae derived from using the images of the conductors—and, therefore, obtaining accurate results through the use of electronic calculators.

   The value of the complex penetration depth in m is given by
   \[
   \bar{\varphi} = \sqrt{\rho/(3j\mu)},
   \]
   where \( \rho \) is the ground resistivity in \( \Omega \cdot m \), \( j \) is the imaginary number, \( \omega \) is the angular frequency in rad/s—equal to 377 rad/s for 60 Hz—, and \( \mu \) is the ground permeability in H/m. Assuming that the ground permeability is equal to the permeability in free space, \( \mu_0 \),
   \[
   \mu = 4\pi \times 10^{-7} \text{ H/m}.
   \]

   ![Figure 9.11](image)

   **FIGURE 9.11** Transmission line geometry for impedance calculations including complex penetration depth.

2. **Calculate the Self-Impedance** \( Z_{ii} \)

   The self-impedance, \( Z_{ii} \), which is one of the diagonal elements of the series impedance matrix, represents the impedance in \( \Omega/m \) of the loop “conductor \( i \)/ground return.” Such an impedance can be determined from
   \[
   Z_{ii} = R_i + j\omega \frac{\mu_0}{2\pi} \ln \frac{2(h_i+\bar{\varphi})}{\text{GMR}_{ii}},
   \]
   where \( R_i \) is the
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ac resistance\(^1\) of the conductor\(^2\) \(i\) in \(\Omega/m\), \(h_i\) is its average height above ground in m, which can be calculated as tower height \(-\frac{\sqrt{3}}{4}\) sag for spans shorter than 500 m (Dommel, 1992), and GMR\(_i\) is its geometric mean radius in m.

The quantity \(j\omega \frac{\mu_0}{2\pi} \ln \frac{2(h_i + \bar{p})}{\text{GMR}_i}\) has a real part and an imaginary part. The real part will yield the losses in the nonideal ground return. As a result, the self-resistance corrected by ground return \(R_{ii}\), which is the real part of \(Z_{ii}\), will take into account both losses in the conductor \(i\) and losses in the nonideal ground return and will be greater than the resistance of the conductor \(R_i\). The imaginary part, on the other hand, will yield the equivalent self-reactance from which the equivalent self-inductance, \(L_{ii}\), in H/m—the self-inductance corrected by ground return—can be derived as \(L_{ii} = \text{Im}\{\text{Z}_{ii}\}/\omega\).

Similarly, the self-impedance, \(\overline{Z}_{ii}\), can be obtained by replacing the index \(i\) with the index \(k\) in the above-mentioned formulae. If the impedances are needed in \(\Omega/m\), the results obtained in \(\Omega/m\) must be multiplied by 1600.

3. Calculate the Mutual Impedance, \(\overline{Z}_{ik}\)

The mutual impedance, \(\overline{Z}_{ik}\), which is one of the off-diagonal elements of the series impedance matrix, represents the impedance in \(\Omega/m\) between the loops "conductor \(i\)/ground return" and "conductor \(k\)/ground return." Such an impedance can be determined from

\[
\overline{Z}_{ik} = j\omega \frac{\mu_0}{2\pi} \ln \frac{D_{ik}}{D_{ik}'}
\]

where \(D_{ik}\) is the distance between the conductor \(i\) and the conductor \(k\), and \(D_{ik}'\) is the distance between the conductor \(i\) and the image of the conductor \(k\).

Given the above-depicted geometry, \(D_{ik} = \sqrt{(h_i - h_k)^2 + d_{ik}^2}\) and \(D_{ik}' = \sqrt{(h_i + h_k + \bar{p})^2 + d_{ik}^2}\), where \(h_i\) is the average height above ground of the conductor \(i\), \(h_k\) is the average height above ground of the conductor \(k\), and \(d_{ik}\) is the horizontal distance between the conductors \(i\) and \(k\).

The impedance, \(\overline{Z}_{ik}\), has a real part and an imaginary part as well. The real part, \(R_{ik}\), represents the phase shift that shows up in the induced voltage for including the nonideal ground return, and the imaginary part yields the equivalent mutual reactance from which the equivalent mutual inductance, \(L_{ik}\)—the mutual inductance corrected by ground return—can be derived as \(L_{ik} = \text{Im}\{\overline{Z}_{ik}\}/\omega\).

By symmetry, \(\overline{Z}_{ki} = \overline{Z}_{ik}^*\).

4. Eliminate Ground Wires

The order of the series impedance matrix can be reduced since the potentials of the ground wires are equal to zero. To do so, let the set of linear equations be split into subsets of ungrounded conductor equations and subsets of ground wire equations, as indicated in Dommel (1992), and the impedance matrix be divided into the ungrounded conductor submatrices \([Z_{uu}]\) and \([Z_{ug}]\) and the ground wire submatrices \([Z_{gu}]\) and \([Z_{gg}]\).

Similarly, let the far-end terminals be short-circuited and voltage drops per unit length

---

\(^1\)The ac resistance increases as frequency steps up due to the skin effect. For an ACSR conductor, at frequencies higher than power frequency, the ac resistance can be calculated by using an equivalent tubular conductor and Bessel functions (Lewis and Tuttle, 1959). Frequency Dependent Parameters of Stranded Conductors can also be calculated with the formulae given in Galloway et al. (1964).

\(^2\)If \(i\) is a bundle of conductors, replace the bundle with one equivalent conductor located at the center of the bundle and use the equivalent resistance of the bundle, the equivalent geometric mean radius of the bundle, and the average height of the bundle instead.
and currents be defined for ungrounded conductors and ground wires by the vectors \([\Delta V_u]\) and \([I_u]\) and \([\Delta V_g]\) and \([I_g]\), respectively:

\[
\begin{bmatrix}
[\Delta V_u] \\
[\Delta V_g]
\end{bmatrix}
= 
\begin{bmatrix}
[Z_{uu}] & [Z_{ug}] \\
[Z_{gu}] & [Z_{gg}]
\end{bmatrix}
\cdot
\begin{bmatrix}
[I_u] \\
[I_g]
\end{bmatrix}
\]

Since the vector of voltage drops across the ground wires \([\Delta V_g] = 0\), the system can be written as \([\Delta V_u] = [Z_{red}] \cdot [I_u]\), where the reduced matrix \([Z_{red}] = [Z_{uu}] - [Z_{ug}][Z_{gg}]^{-1}[Z_{gu}]\) as a result of a Kron’s reduction.

**Related Calculations.** For a balanced transmission line with only one ground wire, the expression \([Z_{red}][Z_{gg}]^{-1}[Z_{gu}]\) becomes

\[
\frac{Z_{ug}}{Z_{gg}} \cdot [U],
\]

where \(Z_{ug}\) is the mutual impedance between the loops “ungrounded conductor/ground return” and “ground wire/ground return,” \(Z_{gg}\) is the self-impedance of the loop “ground wire/ground return,” and \([U]\) is the unit matrix.

Array manipulations can easily be carried out by using matrix-oriented programs such as MATLAB and MATHCAD. Transmission-line parameters can also be calculated through support routines of EMTP-type programs such as LINE CONSTANTS (Dommel, 1992).

**INDUCTIVE SEQUENCE IMPEDANCES OF THREE-PHASE TRANSMISSION LINES**

Consider a three-phase transmission line whose series impedance matrix has been calculated and reduced as explained in the subsection “Inductive Impedances of Multiconductor Transmission Lines Including Ground Return Corrections.” Determine the series sequence parameters of such a line.

**Calculation Procedure**

1. **Premultiply and Multiply \([Z_{abc}]\) by the Transformation Matrices**

The matrix of sequence impedances \([Z_{abc}] = [T]^{-1} \cdot [Z_{abc}] \cdot [T]\), where \([T]\) is the symmetrical component transformation matrix,\(^3\) and \([Z_{abc}]\) is the reduced matrix of ungrounded (phase) conductors that includes the ground return corrections.

\[
[T] = 
\begin{bmatrix}
1 & 1 & 1 \\
1 & a^2 & a \\
1 & a & a^2
\end{bmatrix}
\]

\[
[T]^{-1} = \frac{1}{3} 
\begin{bmatrix}
1 & 1 & 1 \\
1 & a & a^2 \\
1 & a^2 & a
\end{bmatrix}
\]

\(^3\)\(a = e^{j120^\circ}\) and \(a^2 = e^{-j120^\circ}\) in the transformation matrix.
Related Calculations. When the three-phase transmission line is balanced, the matrix of series sequence impedances is a diagonal matrix, as indicated below:

\[
[Z_{012}] = \begin{bmatrix}
Z_0 & 0 & 0 \\
0 & Z_1 & 0 \\
0 & 0 & Z_2
\end{bmatrix}
\]

where the zero sequence impedance \(Z_0\) = \(Z_{self} + 2Z_{mutual}\), the positive sequence impedance \(Z_1\) = \(Z_{self} - Z_{mutual}\), and the negative sequence impedance \(Z_2\) = \(Z_{self} - Z_{mutual}\). \(Z_{self}\) is found by averaging the elements \(Z_{ab}, Z_{ac},\) and \(Z_{bc}\) of \(Z_{abc}\), and \(Z_{mutual}\) by averaging the elements \(Z_{abc}, Z_{bac},\) and \(Z_{cua}\) of \(Z_{abc}\).

INDUCTIVE REACTANCE OF CABLES IN DUCTS OR CONDUIT

Find the reactance per 1000 ft (304.8 m) if three single conductors each having 2-in. (5 cm) outside diameters and 750-cmil cross sections are enclosed in a magnetic conduit.

Calculation Procedure

1. Determine Inductance

   Use \(L = (2 \times 10^{-3})(\frac{1}{4} + \ln(D/a))\) H/m. Common practice dictates that the reactance be given in ohms per thousand feet. Hence, \(X_L = [0.0153 + 0.1404 \log(D/a)]\).

2. Use Nomogram for Solution

   A nomogram based on the preceding equation is provided in Fig. 9.12. Two factors are used to improve accuracy. The equation for \(X_L\) produces a smaller reactance than an open-wire line. If randomly laid in a duct, the value of \(D\) is somewhat indeterminate because the outer insulation does not always touch. Therefore, if cables are not clamped on rigid supports, a multiplying factor of 1.2 is used. Further, if confined in a conduit of magnetic material in random lay, a multiplying factor of approximately 1.5 is used. Figure 9.12 includes a correction table for cables bound together rather than randomly laid. Here, sector refers to a cable whose three conductors approximate 120° sectors.

3. Determine \(X_L\) Graphically

   Draw a line from 750 MCM to 2-in spacing and read the inductive reactance of 0.038 \(\Omega\) per thousand feet. Then, \(X_L = (1.5)(0.038) = 0.057 \Omega\) per thousand feet = \(0.19 \times 10^{-3} \Omega/m\).

Related Calculations. For a three-phase cable having concentric stranded conductors with a total of 250 MCM each and a diameter of 0.89 in (2.225 cm), the nomogram yields \(X_L = 0.0315 \Omega\) per thousand feet \(10^{-4} \Omega/m\). If the cable is in a magnetic conduit, the tabulated correction factor is used. Thus, \(X_L = (1.49)(0.0315) = 0.0362 \Omega\) per thousand feet, or \(1.2 \times 10^{-4} \Omega/m\).
Consider a transmission system consisting of \( n \) buried single-circuit cables, in which each single circuit is made up of a high-voltage conductor and a cable sheath, and in which, for simplicity, only three single circuits, \( a \), \( b \), and \( c \), have been depicted. Determine the self and mutual series impedances of such a system taking into account the ground return (Fig. 9.13).

**Calculation Procedure**

1. **Calculate the Loop Impedances**

   The method and the formulae proposed in Wedepohl and Wilcox (1973) and Dommel (1992) are appropriate to describe the electric quantities associated with this cable system. Such a formulation takes into account the skin effect in the conductors, approximates the ground return corrections proposed in Pollaczek (1931), which are the appropriate corrections for underground cables, to closed-form expressions, and allows system studies, if desired, at frequencies higher than power frequency. By assuming \( n = 3 \) and that the far-end terminals are short-circuited, six coupled equations describe the loop quantities associated with the cable system here under study.
\[
\begin{bmatrix}
\Delta V_{1a} \\
\Delta V_{2a} \\
\Delta V_{1b} \\
\Delta V_{2b} \\
\Delta V_{1c} \\
\Delta V_{2c}
\end{bmatrix} = \begin{bmatrix}
\bar{Z}_{11a} & \bar{Z}_{12a} & 0 & 0 & 0 & 0 \\
\bar{Z}_{11a} & \bar{Z}_{22a} & \bar{Z}_{ab} & 0 & \bar{Z}_{ac} & \bar{Z}_{ac} \\
0 & 0 & \bar{Z}_{11b} & \bar{Z}_{12b} & 0 & 0 \\
0 & \bar{Z}_{ab} & \bar{Z}_{12b} & \bar{Z}_{22b} & 0 & \bar{Z}_{ac} \\
0 & 0 & 0 & 0 & \bar{Z}_{11c} & \bar{Z}_{12c} \\
0 & \bar{Z}_{ac} & 0 & \bar{Z}_{bc} & \bar{Z}_{12c} & \bar{Z}_{22c}
\end{bmatrix} \begin{bmatrix}
\bar{I}_{1a} \\
\bar{I}_{2a} \\
\bar{I}_{1b} \\
\bar{I}_{2b} \\
\bar{I}_{1c} \\
\bar{I}_{2c}
\end{bmatrix}.
\]

where:

\[
\Delta \bar{V}_1 = \Delta \bar{V}_{\text{core}} - \Delta \bar{V}_{\text{sheath}}
\]

\[
\Delta \bar{V}_2 = \Delta \bar{V}_{\text{sheath}}
\]

\[
\bar{I}_1 = \bar{I}_{\text{core}}
\]

\[
\bar{I}_2 = \bar{I}_{\text{sheath}} + \bar{I}_{\text{core}}
\]

\[
\bar{Z}_{11} = \bar{Z}_{\text{core-out}} + \bar{Z}_{\text{core/sheath-insulation}} + \bar{Z}_{\text{sheath-in}}
\]

\[
\bar{Z}_{22} = \bar{Z}_{\text{sheath-out}} + \bar{Z}_{\text{sheath-earth-insulation}} + \bar{Z}_{\text{self earth-return}}
\]

\[
\bar{Z}_{12} = \bar{Z}_{\text{sheath-mutual}}
\]

\[
\bar{Z}_{ab} = \bar{Z}_{\text{mutual earth-return}_{ab}}
\]

\[
\bar{Z}_{ac} = \bar{Z}_{\text{mutual earth-return}_{ac}}
\]

\[
\bar{Z}_{bc} = \bar{Z}_{\text{mutual earth-return}_{bc}}
\]

with:

\[
a, b, c = \text{subscripts denoting quantities associated with the single circuits of phases } a, b, \text{ and } c, \text{ respectively.}
\]

\[
\Delta \bar{V}_{\text{core}} = \text{voltage drop per unit length across core with respect to ground.}
\]

\[
\Delta \bar{V}_{\text{sheath}} = \text{voltage drop per unit length across sheath with respect to ground.}
\]

\[
\bar{I}_{\text{core}} = \text{current flowing through core conductor.}
\]

\[
\bar{I}_{\text{sheath}} = \text{current flowing through sheath.}
\]

\[
\bar{Z}_{\text{core-out}} = \text{internal impedance per unit length of core conductor, calculated from the voltage drop on the outer surface of the core per unit current, when the current returns through the outer conductor. In this case, the outer conductor is the sheath.}
\]

\[
\bar{Z}_{\text{core/sheath-insulation}} = \text{impedance per unit length of insulation between core and sheath.}
\]

\[
\bar{Z}_{\text{sheath-in}} = \text{internal impedance per unit length of sheath, calculated from the voltage drop on the inner surface of the sheath per unit current, when the current returns through the inner conductor. In this case, the inner conductor is the core.}
\]

\[
\bar{Z}_{\text{sheath-out}} = \text{internal impedance per unit length of sheath, calculated from the voltage drop on the outer surface of the sheath per unit current, when the current returns through the outer conductor. In this case, the outer conductor is the earth-return path.}
\]

\[
\bar{Z}_{\text{sheath/earth-insulation}} = \text{impedance per unit length of insulation between sheath and earth-return path.}
\]

\[
\bar{Z}_{\text{sheath-mutual}} = \text{mutual impedance per unit length of sheath. In this case, it is the mutual impedance between the inside loop “core/sheath” and the outside loop “sheath/earth” of one single circuit.}
\]

\[
\bar{Z}_{\text{self earth-return}} = \text{self impedance per unit length of the earth-return path.}
\]
\[ Z_{\text{mutual earth-return}} = \text{mutual impedance per unit length of the earth-return path. In this case, it is the mutual impedance between the outermost loop "sheath/earth" of one single circuit and the outermost loop "sheath/earth" of another single circuit.} \]

\[ Z_{\text{core-out}} = \frac{\rho_{\text{core}} \bar{m}_{\text{core}}}{2\pi r_{\text{core}}} \cosh(0.777/\bar{m}_{\text{core}} r_{\text{core}}) + \frac{0.356 \rho_{\text{core}}}{\pi r_{\text{core}}} \text{ in } \Omega/m. \text{ This formula yields a maximum error of 4 percent in the resistive part—it occurs when } |\bar{m}_{\text{core}}| = 5 — \text{ and a maximum error of 5 percent in the reactive component—it occurs when } |\bar{m}_{\text{core}}| = 3.5 \text{ (Wedepohl and Wilcox, 1973). For other values of } |\bar{m}_{\text{core}}|, \text{ the formula is very accurate and avoids the evaluation of Bessel functions; } r_{\text{core}} \text{ is the radius of the core conductor in m, } \rho_{\text{core}} \text{ is the resistivity of the core conductor in } \Omega \cdot \text{m, and } \bar{m}_{\text{core}} \text{ is the reciprocal of the complex penetration depth of the core; } \bar{m}_{\text{core}} = \sqrt{(\rho_{\text{core}} \mu_{\text{core}}) / \rho_{\text{core}}} \text{ in } \text{m}^{-1}, \text{ where } \mu_{\text{core}} \text{ is the magnetic permeability of the core in H/m, } \mu_{\text{core}} = \mu_{\text{core}}' \mu_{\text{core}}'' \text{ with } \mu_{\text{core}}'' \neq 1 \text{ if the material of the core conductor is magnetic, and } \omega \text{ is the angular frequency in rad/s.} \]

\[ Z_{\text{sheath-in}} = \frac{\rho_{\text{sh}}}{2\pi r_{\text{sh-in}}} \left\{ \bar{m}_{\text{sh}} \cosh(\bar{m}_{\text{sh}} \Delta_{\text{sh}}) - \frac{1}{r_{\text{sh-in}} + r_{\text{sh-out}}} \right\} \text{ in } \Omega/m. \text{ This formula yields a good accuracy if the condition: } \frac{r_{\text{sh-out}} - r_{\text{sh-in}}}{r_{\text{sh-out}} + r_{\text{sh-in}}} < \frac{1}{8} \text{ is satisfied (Wedepohl and Wilcox, 1973); } r_{\text{sh-out}} \text{ is the outer radius of the sheath in m, } r_{\text{sh-in}} \text{ is the inner radius of the sheath in m, } \rho_{\text{sh}} \text{ is the resistivity of the sheath in } \Omega \cdot \text{m, and } \bar{m}_{\text{sh}} \text{ is the reciprocal of the complex penetration depth of the sheath; } \bar{m}_{\text{sh}} = \sqrt{(\rho_{\text{sh}} \mu_{\text{sh}}) / \rho_{\text{sh}}} \text{ in } \text{m}^{-1}, \text{ where } \mu_{\text{sh}} \text{ is the magnetic permeability of the sheath in H/m, } \mu_{\text{sh}} = \mu_{\text{sh}}' \mu_{\text{sh}}'' \text{ with } \mu_{\text{sh}}'' \neq 1 \text{ if the material of the sheath is magnetic, and } \Delta_{\text{sh}} \text{ is the thickness of the sheath in m, which can be calculated as } (r_{\text{sh-out}} - r_{\text{sh-in}}). \]

\[ Z_{\text{sheath-out}} = \frac{\rho_{\text{sh}}}{2\pi r_{\text{sh-out}}} \left\{ \bar{m}_{\text{sh}} \cosh(\bar{m}_{\text{sh}} \Delta_{\text{sh}}) - \frac{1}{r_{\text{sh-in}} + r_{\text{sh-out}}} \right\} \text{ in } \Omega/m. \text{ This formula yields a good accuracy if the condition between radii mentioned for } Z_{\text{sheath-in}} \text{ is satisfied.} \]

\[ Z_{\text{sheath-mutual}} = \frac{\rho_{\text{sh}} \bar{m}_{\text{sh}}}{\pi (r_{\text{sh-in}} + r_{\text{sh-out}})} \cosech(\bar{m}_{\text{sh}} \Delta_{\text{sh}}) \text{ in } \Omega/m. \text{ This formula yields a good accuracy if the condition between radii mentioned for } Z_{\text{sheath-in}} \text{ and } Z_{\text{sheath-out}} \text{ is satisfied.} \]

\[ Z_{\text{core/sheath-insulation}} = \frac{j \mu_{\text{sh}}}{2\pi} \ln \left( \frac{r_{\text{sh-in}}}{r_{\text{core}}} \right) \text{ in } \Omega/m, \text{ where } \mu_{\text{sh}} \text{ is the magnetic permeability of the insulation between core and sheath in H/m.} \]

\[ Z_{\text{sheath/earth-insulation}} = \frac{j \mu_{\text{sh}}}{2\pi} \ln \left( \frac{R}{r_{\text{sh-out}}} \right) \text{ in } \Omega/m, \text{ where } \mu_{\text{sh}} \text{ is the magnetic permeability of the insulation between sheath and earth in H/m, and } R \text{ is the outside radius of the outermost insulation of the cable in m.} \]

\[ Z_{\text{self earth-return}} = \frac{j \mu_{\text{sh}}}{2\pi} \left\{ -\ln \left( \frac{\gamma \bar{m}_{\text{sh}} R}{2} \right) + \frac{1}{2} - \frac{1}{2} \bar{m}_{\text{sh}} \right\} \text{ in } \Omega/m. \text{ This formula yields very accurate results at frequencies for which } |\bar{m}_{\text{sh}}| < 0.25 \text{ (Wedep-}
The conductor quantities indicated below: row 6 to row 5. By doing so, it is possible to prove that the system is described through coupled equations and include the corresponding impedances \( Z_{bc} \) and \( Z_{ac} \) into the single loops. (Smith and Barger, 1972).

If the single circuits have additional conductors, for example, armors, add three more coupled equations and include the corresponding impedances \( Z_{ab} \) and \( Z_{ac} \) into the single loops. (Smith and Barger, 1972). Move also the impedances \( Z_{ab}, Z_{ac}, Z_{bc} \), and \( Z_{cc} \); since the outermost loops will be the ones “armor/earth.” Derive the formulae for the new impedances by analogy and take care of utilizing the right electric properties as well as the appropriate radii.

2. Transform the Loop Quantities into Conductor Quantities

To transform the loop quantities into conductor quantities, use the procedure recommended in Dommel (1992) as follows. Add row 2 to row 1, add row 4 to row 3, and add row 6 to row 5. By doing so, it is possible to prove that the system is described through the conductor quantities indicated below:

\[
\begin{bmatrix}
\Delta v_{\text{core}a} \\
\Delta v_{\text{sheath}a} \\
\Delta v_{\text{core}b} \\
\Delta v_{\text{sheath}b} \\
\Delta v_{\text{core}c} \\
\Delta v_{\text{sheath}c}
\end{bmatrix}
= \begin{bmatrix}
Z_{ab} & Z_{ac} & Z_{bc} \\
Z_{ab} & Z_{ad} & Z_{bd} \\
Z_{ab} & Z_{ad} & Z_{bd} \\
Z_{ac} & Z_{ad} & Z_{bd} \\
Z_{ac} & Z_{ad} & Z_{bd} \\
Z_{ac} & Z_{ad} & Z_{bd}
\end{bmatrix}
\begin{bmatrix}
\Delta i_{\text{core}a} \\
\Delta i_{\text{sheath}a} \\
\Delta i_{\text{core}b} \\
\Delta i_{\text{sheath}b} \\
\Delta i_{\text{core}c} \\
\Delta i_{\text{sheath}c}
\end{bmatrix}
\]

where \( Z_{ac} = Z_{a1} + 2Z_{a2} + Z_{a3} \), \( Z_{ab} = Z_{b1} + Z_{b2} \), and \( Z_{ad} = Z_{d2} \). The diagonal elements \( Z_{cc} \) and \( Z_{ss} \) are the self-impedances of the core and sheath with return through earth, respec-

\[
\overline{Z}_{\text{mutual earth-return}} = \frac{3j\mu}{2\pi} \left\{ -\ln\left( \frac{\gamma m d}{2} \right) + \frac{j_0}{\sqrt{2}} \frac{m \ell}{\mu} \right\} \text{ in}\ \Omega/m.
\]

This formula yields very accurate results at frequencies for which \(|\frac{m \ell}{\mu}| < 0.25 \) (Wedepohl and Wilcox, 1973); \( d \) is the distance between single circuits \( a \) and \( b \) for \( Z_{ab} \), the distance between single circuits \( a \) and \( c \) for \( Z_{ac} \), and the distance between single circuits \( b \) and \( c \) for \( Z_{bc} \); \( \ell \) is the sum of the depths of the single circuits \( a \) and \( b \) for \( Z_{ab} \), the sum of the depths of the single circuits \( a \) and \( c \) for \( Z_{ac} \), and the sum of the depths of the single circuits \( b \) and \( c \) for \( Z_{bc} \). Errors for \( \overline{Z}_{\text{self earth-return}} \) and \( \overline{Z}_{\text{mutual earth-return}} \) are lower than 1 percent up to frequencies of 100 kHz (Dommel, 1992). The formula is also very accurate if the depths at which the cables are buried are close to 1 m (Wedepohl and Wilcox, 1973).

Related Calculations. If the impedances are needed in \( \Omega \) per 1000 ft, multiply the impedance in \( \Omega/m \) by 304.8 m.

If the cable has multiwire concentric neutral conductors, replace the neutral wires with an equivalent concentric sheath and assume that the thickness of such a sheath is equal to the diameter of one of the neutral wires. All neutral wires are assumed to be identical (Smith and Barger, 1972).

If the single circuits have additional conductors, for example, armors, add three more coupled equations and include the corresponding impedances \( Z_{21} \) and \( Z_{11} \) into the single loops. (Smith and Barger, 1972). Move also the impedances \( Z_{ab}, Z_{ac}, Z_{bc} \), and \( Z_{cc} \); since the outermost loops will be the ones “armor/earth.” Derive the formulae for the new impedances by analogy and take care of utilizing the right electric properties as well as the appropriate radii.
The off-diagonal elements \( Z_{cs}, Z_{ab}, Z_{ac}, \) and \( Z_{bc} \) are the mutual impedances between core and sheath of one cable with return through earth, between sheath \( a \) and sheath \( b \) with return through earth, between sheath \( a \) and sheath \( c \) with return through earth, and between sheath \( b \) and sheath \( c \) with return through earth, respectively. As a result of the above-mentioned arithmetic operations, the system is represented in nodal form, with currents expressed as conductor currents and voltages expressed as voltage drops across the conductors with respect to ground.

**Related Calculations.** If armors are present, add rows 2 and 3 to row 1 and add row 3 to row 2. Add also rows 5 and 6 to row 4 and add row 6 to row 5. Similarly, add rows 8 and 9 to row 7 and add row 9 to row 8.

Array manipulations can easily be carried out by using matrix-oriented programs such as MATLAB and MATHCAD. Underground cable parameters can also be calculated through support routines of EMTP-type programs such as CABLE CONSTANTS and CABLE PARAMETERS (Ametani, 1980).

### 3. Eliminate the Sheaths

By interchanging the corresponding rows and columns in the impedance matrix, move voltage drops across sheaths and current flows through sheaths to the bottom of the vectors of voltages and currents, respectively.

Then let the set of linear equations be split into subsets of core conductor equations and subsets of sheath equations, and the impedance matrix be divided into the core conductor submatrices \([Z_{cc}]\) and \([Z_{es}]\) and the sheath submatrices \([Z_{sc}]\) and \([Z_{ss}]\). Similarly, let the far-end terminals be short-circuited and voltage drops per unit length and currents be defined for core conductors and sheaths by the vectors \([\Delta V_c]\) and \([I_c]\) and \([\Delta V_s]\) and \([I_s]\), respectively:

\[
\begin{bmatrix}
[\Delta V_c] \\
[\Delta V_s]
\end{bmatrix} = 
\begin{bmatrix}
[Z_{cc}] & [Z_{cs}] \\
[Z_{sc}] & [Z_{ss}]
\end{bmatrix} 
\cdot 
\begin{bmatrix}
[I_c] \\
[I_s]
\end{bmatrix} 
\]

Since the vector of voltage drops across the sheaths \([\Delta V_s] = 0\), assuming that both terminals of each sheath are grounded, the system can be written as \([\Delta V_c] = [Z_{red}]\cdot[I_c]\), where the reduced matrix \([Z_{red}] = [Z_{cc}] - [Z_{cs}][Z_{ss}]^{-1}[Z_{sc}]\) as a result of a Kron’s reduction.

### INDUCTIVE SEQUENCE IMPEDANCES OF THREE-PHASE UNDERGROUND CABLES

Consider a three-phase underground transmission system whose series impedance matrix has been calculated and reduced as explained in the subsection “Inductive Impedances of Multiconductor Underground Cables including Ground Return Corrections.” Determine the series sequence parameters of such a system.

#### Calculation Procedure

1. **Premultiply and Multiply \([Z_{abc}]\) by the Transformation Matrices**

The matrix of sequence impedances \([Z_{012}] = [T]^{-1} \cdot [Z_{abc}] \cdot [T]\), where \([T]\) is the symmetrical component transformation matrix and \([Z_{abc}]\) is the reduced matrix of core conductors that includes the sheaths and the ground return corrections.

**Related Calculations.** Simple formulae for the calculation of sequence impedances of cables, including Carson’s ground return corrections, are given in Lewis and Allen (1978). Formulae of sequence impedances of cables are also given in Westinghouse Electric Corporation (1964). For the calculation of sequence impedances of pipe-type cables, see Neher (1964).
CAPACITANCE ASSOCIATED WITH TRANSMISSION LINES

Determine the balanced charging current fed from one end to a 230-kV, three-phase transmission line having a capacitive reactance of 0.2 MΩ·mi/phase (0.32 MΩ·km phase). The line is 80 mi (128.7 km) long.

Calculation Procedure

1. Determine Capacitive Reactance

The total capacitive reactance per phase, which is assumed to shunt each phase to ground, is $X_C = 0.32/128.7 = 0.0025$ MΩ.

2. Calculate Charging Current

For the voltage-to-neutral value of $230/\sqrt{3} = 133$ kV, the charging current $I_c$ is $(133 \times 10^3)/(0.0025 \times 10^6) = 53.2$ A.

CAPACITANCE OF TWO-WIRE LINE

Determine the capacitance of a long, round conductor carrying a uniform charge density $\rho_c$ on its outer surface (surplus charge always migrates to the outer surface of any conductor). The conductor is surrounded by an outward (for positive charge) vectorial electric field that appears to radiate from the center of the conductor, although it originates with $\rho_c$ on the surface.

Calculation Procedure

1. Determine the Electric Potential

The magnitude of the electric field intensity $E$ is given by: $E = \rho_c/(2\pi\epsilon r)$, where $\epsilon$ is the permittivity. In free space, $\epsilon = 10^{-9}/36\pi$ F/m. For consistency, the distance $r$ from the center of the conductor is in meters and $\rho_c$ is in coulombs per meter. Integration of $E$ yields the electric potential $V$ between points near the conductor (Fig. 9.14): $V_{ab} = (\rho_c/2\pi\epsilon)\ln(b/a)$ V. The notation $V_{ab}$ indicates that the voltage is the potential at point $a$ with respect to point $b$.

2. Consider a Two-Conductor Line

Consider the two conductors as forming a long, parallel conductor system (Fig. 9.15). Each conductor has an equal but opposite charge, typical of two-wire transmission systems. Further, it is assumed that the charge density per unit area is uniform in each conductor, even though a charge attraction exists between conductors, making it nonuniform. This assumption is completely adequate for open-wire lines for which $D \gg a$. 
Because the conductors bear charges of opposite polarities, the electric field at point \( r \) in the plane of the conductors between them is \( E = (p_0/2\pi\varepsilon)[-1/(r - 1/(D - r))] \) \( \text{V/m} \), where \( r \) is the distance from the center of conductor 1 \( (r \approx a) \), and \( D \) is the center-to-center spacing between conductors. By integrating \( E \), potential \( V_1 \) at conductor 1 with respect to point \( r \) is obtained: 
\[
V_1 = (p_0/2\pi\varepsilon)\ln[r(D - a)/(a(D - r))] \text{ V}.
\]
If \( r \) extends to conductor 2 and \( D \approx a \), the potential of conductor 1 with respect to conductor 2 is: 
\[
V_{12} = (p_0/2\pi\varepsilon)\ln(D/a) \text{ V}.
\]

3. Calculate Capacitance

The capacitance per unit length is determined from 
\[
C = q/l \text{ F/m},
\]
where \( q = p_0l \) and \( l \) is the total line length. Dividing by \( l \) yields 
\[
C = C/l = \pi\varepsilon\ln(D/a) \text{ F/m},
\]
which is the capacitance between conductors per meter length.

4. Determine Capacitance to Vertical Plane between Conductors at \( D/2 \)

The potential for conductor 1 with respect to this plane (or neutral) is 
\[
V_n = (p_0/2\pi\varepsilon)\ln(D/a) \text{ V}.
\]
If \( D \approx a \), the capacitance to neutral is then 
\[
C = (2\pi\varepsilon)/(\ln(D/a)) \text{ F/m or 0.0388/log}(D/a) \mu\text{F/mi}.
\]

CAPACITIVE REACTANCE OF TWO-WIRE LINE

Find the capacitive reactance to neutral for a two-conductor transmission line if \( D = 8 \text{ ft} \) (2.4 m), \( a = 0.25 \text{ in.} \) (0.00625 m), and the length of the line is 10 \text{ mi} (16 km). The frequency is 377 rad/s.

Calculation Procedure

1. Calculate Capacitive Reactance

Recall that \( X_C = 1/\omega C \). Substituting for \( C = 0.0388 \log(D/a) \), obtain 
\[
X_C = 1/[((377)(0.0388)(10)/\log(2.4/0.00625)] = 0.0026 \text{ M}\Omega \text{ to neutral}.
\]

Related Calculations. This is a large value of shunt impedance and is usually ignored for a line this short. It also follows that the capacitive reactance between conductors is twice the above value.

CAPACITANCE OF THREE-PHASE LINES

Determine the capacitance for a three-phase line.

Calculation Procedure

1. Consider Capacitance to Neutral

The capacitance to neutral of a three-phase transmission line is best established by considering equilateral spacing of the conductors initially. Other spacing that is unsymmetrical is commonly considered using the geometric mean distance in the equilateral case. The error that results is insignificant, especially when consideration is given to the
uncertainties of an actual line stemming from line towers and terrain irregularities.

2. Determine Phase Voltages

As shown in Fig. 9.16, \( V_{an} \), the potential of phase \( a \) with respect to the center of the triangle (the neutral), can be found by superimposing the potentials from all phases along the dimension from phase \( a \) to the center. The net charge is zero for any cross section, as it was for the two-wire line. Also, \( D \gg a \). Superscripts to identify the three-phase potentials along the dimension for \( a \) to \( b \) are necessary. Thus, \( V_{an} = (\rho_1/(2\pi\epsilon))\ln[(D/\sqrt{3})/D] \) due to phase \( a \), \( V_{bn} = (\rho_2/(2\pi\epsilon))\ln[(D/\sqrt{3})/D] \) due to phase \( b \), and \( V_{cn} = (\rho_3/(2\pi\epsilon))\ln[(D/\sqrt{3})/D] \) due to phase \( c \).

The sum of the above three equations yields \( V_{an} \). Also \( \rho_1 + \rho_2 + \rho_3 = 0 \). Thus, \( V_{an} = (\rho_1/(2\pi\epsilon))\ln(D/a) \). This equation has the same form as the equation for the potential to neutral of a two-wire line. The phase-to-neutral potentials of the other phases differ only in phase angle.

3. Determine Capacitance to Neutral

Dividing \( \rho_1 \) by \( |V_{an}| \), we obtain \( C = (2\pi\epsilon)/\ln(D/a) \) F/m = 0.0388/\log(D/a) \( \mu \)F/mi.

CAPACITIVE REACTANCE OF THREE-PHASE LINES

Find the capacitive reactance to neutral of a three-phase line at 377 rad/s (Fig. 9.17). The conductors are ACSR Waxwing and the line is 60 mi (96.6 km) long.

Calculation Procedure

1. Calculate Capacitance

From Table 9.2, the external diameter of Waxwing is 0.609 in (0.015 m). Even though the conductors are not in equilateral spacing, use of GMD produces a sufficiently accurate capacitance to neutral. Therefore, \( \text{GMD} = (6^2)(12)^{1/3} = 7.54 \) m and \( a = 0.015/2 = 0.0075 \) m. Hence, \( C = 0.0388/\log(7.54/0.0075) = 0.0129 \) \( \mu \)F/mi or 0.008 \( \mu \)F/km.

2. Calculate Capacitive Reactance

\( X_C = 1/(377)(0.008 \times 10^{-6})(96.6) = 0.0034 \) M\( \Omega \) to neutral.

CAPACITIVE SUSCEPTANCES OF MULTICONDUCTOR TRANSMISSION LINES

Consider a transmission line consisting of \( n \) straight conductors (Fig. 9.18) in which only the conductors \( i \) and \( k \) and their images below earth surface have been shown for simplicity. Determine the self and mutual capacitive susceptances of such a line.
Calculation Procedure

1. Calculate the Potential Coefficient Matrix

The diagonal elements $P_{ii}$ and the off-diagonal elements $P_{ik}$ in m/F of the potential coefficient matrix $[P]$ can be determined from

$$P_{ii} = \frac{1}{2\pi\varepsilon_0} \ln \frac{2h_i}{r_i}$$

and

$$P_{ik} = \frac{1}{2\pi\varepsilon_0} \ln \frac{D_{ik}}{d_{ik}},$$

respectively (Dommel, 1985), where $h_i$ is the average height above ground of the conductor $i$, $r_i$ is the external radius of the conductor $i$, $D_{ik}$ is the distance between the conductor $i$ and the image below earth surface of the conductor $k$, $d_{ik}$ is the distance between the conductors $i$ and $k$, and $\varepsilon_0$ is the permittivity in free space, which is equal to $8.854 \times 10^{-12}$ F/m. 

2. Eliminate the Ground Wires

The potential coefficient matrix $[P]$ can be reduced to a matrix $[P_{\text{rrd}}]$ by applying a Kron’s reduction. To do so, use the same procedure explained in subsection “Impedances of Multiconductor Transmission Lines Including Ground Return Corrections.”

3. Calculate the Capacitance Matrix

Find $[C_{\text{rrd}}]$ in F/m by inverting $[P_{\text{rrd}}]$ since $[C_{\text{rrd}}] = [P_{\text{rrd}}]^{-1}$. However, if the capacitances associated with the ground wires are required, invert $[P]$ to obtain $[C]$.

The capacitance matrices are in nodal form. As a result, the diagonal element $C_{ii}$ stores the sum of the shunt capacitances between conductor $i$ and all other conductors, including ground, and the off-diagonal element $C_{ik}$ stores the negative of the shunt capacitance between conductors $i$ and $k$, as stated in Dommel (1992).

4. Calculate the Capacitive Susceptance Matrix

Find the reduced matrix of capacitive susceptances $[B_{\text{rrd}}]$ in $\Omega^{-1}$/m by multiplying $[C_{\text{rrd}}]$ by $\omega$. If the susceptances are needed in $\Omega^{-1}$/mile, multiply the results obtained in $\Omega^{-1}$/m by 1600.

**CAPACITIVE SEQUENCE SUSCEPTANCES OF THREE-PHASE TRANSMISSION LINES**

Consider a three-phase transmission line whose shunt susceptance matrix has been calculated and reduced as explained in the subsection “Capacitive Susceptances of Multiconductor Transmission Lines.” Determine the shunt sequence parameters of such a line.

---

*If $i$ is a bundle of conductors, replace the bundle with one equivalent conductor located at the center of the bundle and use the equivalent external radius of the bundle and the average height of the bundle instead.*
Calculation Procedure

1. Premultiply and Multiply \( [B_{abc}] \) by the Transformation Matrices

The matrix of shunt sequence susceptances \( [B_{012}] = [T]^{-1} \cdot [B_{abc}] \cdot [T] \), where \( [T] \) is the symmetrical component transformation matrix, and \( [B_{abc}] \) is the reduced matrix of capacitive susceptances that includes the effect of the ground wires.

Related Calculations. When the three-phase transmission line is balanced, the matrix of shunt sequence susceptances is a diagonal matrix as indicated below:

\[
[B_{012}] = \begin{bmatrix}
B_0 & 0 & 0 \\
0 & B_1 & 0 \\
0 & 0 & B_2
\end{bmatrix}
\]

where the zero-sequence susceptance \( B_0 = B_{self} + 2B_{mutual} \), the positive-sequence susceptance \( B_1 = B_{self} - B_{mutual} \), and the negative-sequence susceptance \( B_2 = B_{self} - B_{mutual} \). \( B_{self} \) is found by averaging the elements \( B_{a0} \), \( B_{b0} \), and \( B_{c0} \) of \( [B_{abc}] \), and \( B_{mutual} \) by averaging the elements \( B_{ab} \), \( B_{bc} \), and \( B_{ca} \) of \( [B_{abc}] \).

CAPACITIVE SUSCEPTANCES ASSOCIATED WITH UNDERGROUND CABLES

Consider again the underground cable system described in subsection “Inductive Impedances of Multiconductor Underground Cables Including Ground Return Corrections.” Determine the capacitive susceptances of such a system.

1. Calculate the Self and Mutual Susceptances

The method and the formulae proposed in Wedepohl and Wilcox (1973) and Dommel (1992) are also appropriate to calculate the shunt susceptances of this cable system. By assuming that there is no capacitive coupling among the three phases because of shielding effects, the following six nodal equations can be written:

\[
\begin{bmatrix}
\Delta l_{core,a} \\
\Delta l_{sheath,a} \\
\Delta l_{core,b} \\
\Delta l_{sheath,b} \\
\Delta l_{core,c} \\
\Delta l_{sheath,c}
\end{bmatrix} =
\begin{bmatrix}
B_{cya} & B_{cya} & 0 & 0 & 0 \\
B_{cyb} & B_{cyb} & 0 & 0 & 0 \\
0 & 0 & B_{cyc} & 0 & 0 \\
0 & 0 & B_{cyc} & 0 & 0 \\
0 & 0 & 0 & B_{cya} & B_{cya} \\
0 & 0 & 0 & B_{cya} & B_{cya}
\end{bmatrix}
\begin{bmatrix}
\Delta v_{core,a} \\
\Delta v_{sheath,a} \\
\Delta v_{core,b} \\
\Delta v_{sheath,b} \\
\Delta v_{core,c} \\
\Delta v_{sheath,c}
\end{bmatrix}
\]

where:

- \( a, b, c = \) subscripts denoting quantities associated with the single circuits of phases \( a, b, \) and \( c \), respectively.
- \( \Delta l_{core} = \) charging current per unit length flowing through core conductor.
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\[ \Delta I_{\text{sheath}} = \text{charging current per unit length flowing through sheath.} \]
\[ V_{\text{core}} = \text{voltage of core conductor with respect to ground.} \]
\[ V_{\text{sheath}} = \text{voltage of sheath with respect to ground.} \]
\[ B_{ss} = \text{shunt self-susceptance per unit length of sheath.} \]
\[ B_{cs} = \text{shunt mutual susceptance per unit length between core conductor and sheath.} \]
\[ B_{cc} = \text{shunt self-susceptance per unit length of core conductor.} \]

with:
\[ B_{1} = \text{capacitive susceptance per unit length of insulation layer between core and sheath.} \]
\[ B_{2} = \text{capacitive susceptance per unit length of insulation layer between sheath and earth.} \]
\[ B_{i} = \omega C_{i} \text{ in } \Omega^{-1}/m \text{ and } C_{i} = 2\pi \varepsilon_{i} \varepsilon_{0} / \ln(r_{i}/q_{i}), \text{ where } C_{i} \text{ is the shunt capacitance of the tubular insulation in F/m, } q_{i} \text{ is the inside radius of the insulation, } r_{i} \text{ is the outside radius of the insulation, and } \varepsilon_{0} \text{ is the relative permittivity of the insulation material.} \]

2. **Eliminate the Sheaths**

The matrix of self and mutual susceptance can be reduced by applying a Kron's reduction. To do so, use the same procedure explained in the subsection “Inductive Impedances of Multiconductor Underground Cables Including Ground-Return Corrections.”

3. **Calculate the Sequence Susceptances**

The matrix of capacitive sequence susceptances can be obtained by applying the symmetrical component transformation matrices to the reduced matrix of self and mutual susceptances. To do so, use the same procedure explained in the subsection “Inductive Sequence Impedances of Three-Phase Underground Cables.”

### CHARGING CURRENT AND REACTIVE POWER

Determine the input charging current and the charging apparent power in megavars for the example in Fig. 9.17 if the line-to-line voltage is 230 kV.

**Calculation Procedure**

1. **Calculate Charge Current, \( I_{c} \)**
   \[ I_{c} = \left( \frac{V}{\sqrt{3}} \right) X_{c}, \text{ where } V \text{ is the line-to-line voltage. Then, } I_{c} = (230 \times 10^{3})/(\sqrt{3}) (0.0034 \times 10^{6}) = 39.06 \text{ A.} \]

2. **Calculate Reactive Power, \( Q \)**
   \[ Q = \sqrt{3} V I = (\sqrt{3})(230 \times 10^{3})(39.06) = 15.56 \text{ MVAR for the three-phase line.} \]
Short transmission lines (up to 80 km (50 mi)) are represented by their series impedance consisting of the line resistance $R_L$ and inductive reactance $X_L$. In cases where $R_L$ is less than 10 percent of $X_L$, $R_L$ is sometimes ignored.

The model for medium lines (up to 320 km (200 mi)) is represented in Fig. 9.19, where the line capacitance $C_L$ is considered. Expressions for admittance $Y_L$ and impedance $Z_L$ are:

$$Y_L = \frac{j\omega C_L}{1 \text{ km}} \cdot 1 = \frac{3.77(0.01)(10^{-6})}{320 \text{ km}} \cdot 320 \text{ km} = j1206 \text{ microsiemens (}\mu\text{S})$$

$$Z_L = \frac{R_L}{1 \text{ km}} \cdot 1 + \frac{X_L}{1 \text{ km}} \cdot 1 = \frac{0.2 \Omega}{(320 \text{ km})} + \frac{3.77(2)(10^{-3}) \Omega}{320 \text{ km}} \cdot 320 \text{ km}$$

$$= -64 + j241.3 \Omega$$

For long transmission lines, $V_s$ is the receiving-end voltage and $I_s$ is the receiving-end current. The above equations may be written as: $V_s = AV_r + BI_r$ and $I_s = CV_R + DI_R$, where $A = D = Z_L Y_L/2 + 1$, $B = Z_L$, and $C = Y_L + Z_L Y_L/4$.

For long transmission lines, $V_s = (V_R + I_R Z_c) e^{\gamma/2} + (V_R - I_R Z_c) e^{-\gamma/2}$ and $I_s = (V_s/Z_c + I_Q) e^{\gamma/2} + (V_s/Z_c - I_Q) e^{-\gamma/2}$, where the characteristic impedance $Z_c = \sqrt{Z_Y} \Omega$ and the propagation constant $\gamma = \sqrt{Z_Y}$ per kilometer (or per mile).

**MEDIUM TRANSMISSION-LINE MODELS FOR POWER-FREQUENCY STUDIES**

Calculate the sending-end voltage and current for a 320-km (200 mi) transmission line. The receiving-end line-to-line voltage is 230 kV and the current is 200 A, at a power factor of 0.8 lagging. The line parameters per kilometer are: $R = 0.2 \Omega$, $L = 2 \text{ mH}$, $C = 0.01 \mu\text{F}$, and $f = 60 \text{ Hz}$.

**Calculation Procedure**

1. **Determine $Y_L$ and $Z_L$**

   $$Y_L = \frac{j\omega C_L}{1 \text{ km}} \cdot 1 = \frac{3.77(0.01)(10^{-6})}{320 \text{ km}} \cdot 320 \text{ km} = j1206 \text{ microsiemens (}\mu\text{S})$$

   $$Z_L = \frac{R_L}{1 \text{ km}} \cdot 1 + \frac{X_L}{1 \text{ km}} \cdot 1 = \frac{0.2 \Omega}{(320 \text{ km})} + \frac{3.77(2)(10^{-3}) \Omega}{320 \text{ km}} \cdot 320 \text{ km}$$

   $$= -64 + j241.3 \Omega$$

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Transmission line models for electromagnetic transient studies can be found in Dommel (1992).
2. Calculate A, B, C, and D

\[ A = D = Y_L Z_L / 2 + 1 = j1206(64 + j241.3)/2 + 1 = 0.8553/2.5869^\circ \]

\[ B = Z_L = 64 + j241.3 = 249.64/75.1455^\circ \]

\[ C = Y_L + Z_L V_L^2/4 = j1206 + (64 + j241.3)(j1206/4^2/4 = 0.0011/91.1926^\circ \]

3. Calculate \( V_S \) and \( I_S \)

The receiving-end phase voltage is 230\( \sqrt{3} \) = 132.8 kV.

The sending-end voltage \( V_S \) = (0.8553/2.5864\(^\circ\))(132.8 \times 10^3/4^0) + (249.64/75.1455\(^\circ\))(200/ - 36.9^\circ) = 156.86/13.2873 kV. The magnitude of the sending-end line voltage is 271.69 kV.

The calculation shows \( I_S \) is 147.77 A, which is less than the receiving-end current. (This effect is due to the reactive compensation introduced by the line capacitances).

Related Calculations. In case the sending-end values \( V_S \) and \( I_S \) are known, the receiving-end voltage and current may be calculated by:

\[ V_R = \frac{D V_S - B I_S}{A D - B C} \]

\[ I_R = \frac{-C V_S + A I_S}{A D - B C} \]

LONG TRANSMISSION-LINE MODELS FOR POWER-FREQUENCY STUDIES

Recompute the medium-line model example (320 km long) with the same values of potential, current, and line parameters using the long-line model.

Calculation Procedure

1. Calculate \( Z \) and \( Y \)

Because \( R = 0.2 \Omega/km \) and \( L = 2 \mu H/km \), the series impedance per unit length is \( Z = 0.2 + j0.754 = 0.78/75.1^\circ \) \( \Omega/km \). Since \( C = 0.01 \mu F/km \), the shunt admittance per kilometer is \( Y = j3.77 \) \( S/km \).

2. Calculate \( Z_c \)

The characteristic (surge) impedance is \( Z_c = [0.78/75.1^\circ(3.77 \times 10^{-6}/90^\circ)]^{1/2} = 455/-7.45^\circ \) \( \Omega \). If the resistance is less than one-tenth the inductive reactance per unit length, the characteristic impedance approaches a real number.

3. Calculate the Propagation Constant, \( \gamma \)

\( \gamma = (0.78/75.1^\circ)(3.77 \times 10^{-6}/90^\circ)]^{1/2} = 1.72 \times 10^{-3}/82.55^\circ \). A small resistance results in a value approaching an imaginary number. To be useful, \( \gamma \) must be in rectangular form. Thus, \( \gamma = 0.223 \times 10^{-3} + j1.71 \times 10^{-3} \).

The real part of \( \gamma \) is the attenuation factor \( \alpha \). \( \alpha = 0.223 \times 10^{-3} \) nepers/km. The imaginary part is the phase-shifting constant \( \beta \). \( \beta = 1.71 \times 10^{-3} \) rad/km.
4. Calculate $V_S$ and $I_S$

The per-phase receiving voltage to neutral is 132.8 kV. Substituting the above values in the equations for $V_S$ and $I_S$ yields: $V_S = [(132.8 \times 10^3/[0^\circ)]/2] + [(-0.223 \times 10^{-3})/(200)] [\exp(j1.71 \times 10^{-3})(200)] + [(-0.223 \times 10^{-3})(200)] [\exp(j1.71 \times 10^{-3})(200)] + [(200/455/7.45)] [\exp(0.223 \times 10^{-3})(200)] [\exp(j1.71 \times 10^{-3})(200)] + [(132.8 \times 10^3/[0^\circ)]/2] [\exp(0.223 \times 10^{-3})(200)] [\exp(j1.71 \times 10^{-3})(200)] + [(200/36.9)(455/-7.45)] [\exp(0.223 \times 10^{-3})(200)] [\exp(j1.71 \times 10^{-3})(200)] + [(132.8 \times 10^3/[0^\circ)]/2] [\exp(0.223 \times 10^{-3})(200)] [\exp(j1.71 \times 10^{-3})(200)] + [(200/36.9)(455/-7.45)] [\exp(0.223 \times 10^{-3})(200)] [\exp(j1.71 \times 10^{-3})(200)]. When terms are combined, $V_S = 150.8/8.06^\circ$ kV (phase to neutral) and $I_S = 152.6/−4.52^\circ$ A (line). The magnitude of the line voltage at the input is 261.2 kV. These results correspond to the results of the medium-line model of the previous example with little difference for a 320-km line.

Related Calculations. Equations for $V_S$ and $I_S$ are most easily solved by suitable FORTRAN, C, or C++ algorithms, by EMTP–type programs, or by matrix-oriented programs such as MATLAB and MATHCAD. With such programs, the value of $x$ can be incremented outward from the receiving end to display the behavior of the potential and current throughout the line. Such programs are suitable for lines of any length.

The first term in each equation for $V_S$ and $I_S$ may be viewed as representing a traveling wave from the source to the load end of the line. If $x$ is made zero, the wave is incident at the receiving end. The second term in each equation represents a wave reflected from the load back toward the source. If $x$ is made zero, the value of this wave is found at the receiving end. The sum of the two terms at the receiving end should be 132.8 kV and 200 A in magnitude.

If the impedance at the load end is equal to the characteristic (surge) impedance $Z_c$, the reflected terms (second terms in equations for $V_S$ and $I_S$) are zero. The line is said to be matched to the load. This is hardly possible in transmission line, but is achieved at much higher (e.g., radio) frequencies. This eliminates the so-called standing waves stemming from the summation of terms in the equations for $V_S$ and $I_S$. Such quantities at standing-wave ratio SWR and reflection coefficient $\sigma$ are easily calculated but are beyond the needs of power studies.

### COMPLEX POWER

Determine the complex power $S$ at both ends of the 320-km (200-mi) transmission line using the results of the preceding example.

Calculation Procedure

1. Calculate $S$ at Receiving End

Use $S = 3VI^*$, where $V$ is the phase-to-neutral voltage and $I^*$ is the complex conjugate of the line current under balanced conditions. Then, at the receiving end, $S = (3)(132.8 \times 10^3/[0^\circ])(200)(36.9^\circ) = 63,719 \text{ kW} + j47,841 \text{ kVAR}$. 


2. Calculate $S$ at Sending End

At the sending end, $S = (3)(150.8 \times 10^3/8.06^\circ)(152.6/4.52^\circ) = 67,379$ kW + $j15,036$ kVAR.

Related Calculations. The transmission line must, in this case, be furnishing some of the receiving end’s requirements for reactive power from the supply of stored charge, because the apparent power input in kVAR is less than the output. Power loss of the line may be determined by $Q = 47,841 - 15,036 = 32,905$ kVAR made up by stored line charge and $P = 67,379 - 63,719 = 3660$ kW line-resistance losses.

SURGE IMPEDANCE LOADING

A convenient method of comparing the capability of transmission lines to support energy flow (but not accounting for resistance-loss restrictions) is through the use of surge impedance loading, SIL. If the line is assumed terminated in its own surge impedance value as a load (preferably a real number), then a hypothetical power capability is obtained that can be compared with other lines.

Compare two 230-kV lines for their power capability if $Z_{c1} = 500$ $\Omega$ for line 1 and $Z_{c2} = 400$ $\Omega$ for line 2.

Calculation Procedure

1. Determine Expression for SIL

If $Z_c$ is considered as a load, the load current $I_L$ may be expressed by $I_L = V_t/\sqrt{3}Z_c$ kA, where $V_t$ is the magnitude of the line-to-line voltage in kilovolts. Then, $SIL = \sqrt{3}V_tI_L = V_t^2/Z_c$.

2. Calculate SIL Values

$SIL_1 = 230^2/500 = 106$ MW and $SIL_2 = 230^2/400 = 118$ MW. Line 2 has greater power capability than line 1.

BIBLIOGRAPHY


