Abstract—This paper introduces the heat transfer mechanisms in underground cable installations and analyzes the available solution methods of the diffusion equation. The heat sources and thermal resistances of the different layers of a cable installation are described. The basic concepts behind the Neher-McGrath method (IEEE) are discussed, along with its differences with the IEC standards for underground cable installations. The available commercial computer programs, designed to perform ampacity calculations are listed along with a description of the modeling capabilities of CYME's CYMCAP.

Index Terms—Ampacity, Underground Cables, Neher-McGrath, IEC Standards, CYMCAP, Cables.

I. INTRODUCTION TO CABLE AMPACITY

Ampacity is a term given by Del Mar in 1951 to the current-carrying capacity of a cable. Ampacity in an underground cable system is determined by the capacity of the installation to extract heat from the cable and dissipate it in the surrounding soil and atmosphere. The maximum operating temperature of a cable is a function of the damage that the insulation can suffer as a consequence of high operating temperatures. The insulation withstands different temperatures as function of the duration of the current circulating in the conductors. There are three standardized ampacity ratings: steady state, transient (or emergency) and short-circuit. Only steady state ampacity ratings are discussed in this paper.

Ampacity calculation techniques are as old as the cables themselves. Anders has summarized the history of ampacity calculations in his 1997 book [1]. There are analytical and numerical approaches to calculate cable ampacity. The two major international standard associations, the IEEE and the IEC, have adopted the analytical methods as the basis for their standards [2], [3-9]. The numerical approaches are mainly based on finite differences or finite elements techniques. The finite elements technique is better suited for cable ampacity because of the round geometry of cables.

This paper focuses on the analytical techniques for the computation of cable ampacity in steady-state through the use of assumptions that simplify the problem. For transient (or emergency) calculations the reader is referred to [1], [8], [9], [12] and [13]. Calculation of short-circuit ratings is described in [14] for both adiabatic and non-adiabatic conditions.

II. AN OVERVIEW OF HEAT FLOW

There are three physical mechanisms for heat transfer:

- Conduction
- Convection
- Radiation

Fourier Law describes the heat transferred by conduction. In very simple terms, the heat flux is proportional to the ratio of temperature over space. In an underground cable installation heat conduction occurs everywhere except in the air space in the conduit.

Convection of heat occurs in moving fluids (air, water, etc.) and obeys Newton's Law. The flow of heat is proportional to the temperature difference. In an underground cable installation convection takes place in the air space inside the ducts and at the surface of the earth.

The Stefan-Boltzmann Law describes the radiation of heat phenomenon as being proportional to the difference the temperatures at the power of four \( (t_f^4 - t_0^4) \). In underground cables radiation of heat occurs from the cable(s) to the ducts.

Figure 1 show a typical temperature distribution for a duct bank installation using an engineered backfill on top of the duct bank. From the figure one can appreciate the diffusion of heat that occurs in underground cable systems. Diffusion is a process by which heat is transferred for one region to another in a slow, space-limited fashion described by decaying exponentials. Therefore, there is a practical distance, away from the heat source, beyond which the heating effects are not felt.

Figure 1. Typical temperature distribution of an underground cable installation
III. HEAT SOURCES IN CABLE SYSTEMS

The heat sources in cable installations can be divided into two generic groups: heat generated in conductors and heat generated in insulators. Figure 2 shows a complex cable construction, for illustration purposes, containing many of the possible layers in a cable. The losses in the metallic (conductors) elements are by far the most significant losses in a cable and they are caused by: (a) Joule losses due to impressed currents, circulating currents or induced (eddy current) losses; (b) Hysteresis losses in conductors that are also magnetic.

The following metallic components of a cable system will produce heat:

- Core conductors
- Sheaths
- Concentric neutrals
- Armors
- Skid wires
- Pipes/ducts

The losses in those components are functions of the frequency \( f \) and the temperature \( t \) of operation and proportional to the square of the current \( I \):

\[
W = R_{ac}(f, t) I^2
\]  

(1)

Insulating materials also produce heat. The heat produced in the insulating layers is only important under certain high voltage conditions. The following components could be considered:

- Main insulation
- Shields
- Screens
- Jackets
- Beddings/servings

The loss relationship is given by:

\[
W_d = G_d V^2 = \omega C V^2 \tan(\delta)
\]  

(2)

where \( C \) is the capacitance, \( V \) is the voltage applied and \( \delta \) is the loss angle.

IV. HEAT FLOW IN UNDERGROUND CABLE INSTALLATIONS

In an underground cable system the main heat transfer mechanism is by conduction. With the exception of the air inside the conduits in duct banks or buried ducts installations all the heat is transferred by conduction. Since the longitudinal dimension of a cable is always much larger than the depth of the installation, the problem becomes a two-dimensional heat conduction problem. In Cartesian coordinates one must solve the diffusion equation given by [1]:

\[
\frac{\partial}{\partial x} \left[ \frac{1}{\rho} \frac{\partial t(x, y, t)}{\partial x} \right] + \frac{\partial}{\partial y} \left[ \frac{1}{\rho} \frac{\partial t(x, y, t)}{\partial y} \right] + W = \frac{c}{\rho} \frac{\partial t(x, y, t)}{\partial t}
\]  

(3)

where:

- \( \rho \) = Thermal conductivity of the material
- \( c \) = Volumetric thermal capacity of the material
- \( W \) = Rate of energy (heat) generated

Equation (3) cannot be solved in closed form for the complicated geometry of an underground cable arrangement (see Figure 1). Additionally, numerical solutions could not be obtained in the pre-computer era (before 1950's). However, cables are being installed since the 1890's. Furthermore, since numerical solutions, considering all particularities of the installation, require of the solution of a large number of linear (or nonlinear) equations only with the powerful computers available nowadays, it is becoming practical to get numerical solutions for cable rating purposes.

In view of the complications of the ampacity problem, engineers found practical solutions by combining analytical solutions to simplified geometries with heuristic results. In particular the use of thermal-electrical analogies with empirical work has been very popular with cable engineers.

To that effect, the paper published by Neher and McGrath in 1957 [10] is remarkable; they summarized the knowledge on the ampacity calculation field to that date, and today (2005), the Neher-McGrath method is still being used and it is the base for the IEEE and the IEC standards.

V. THE NEHER-MCGRATH METHOD

The technique known as the Neher-McGrath method for ampacity calculations is based on a thermal-electrical analogy method due to Pashkis and Baker (1942) [11]. The basic idea is to subdivide the study area in layers. Then one substitutes the heat sources by current sources, the thermal resistances by electrical resistances and the thermal capacitances by electrical capacitances. Figure 3 shows the correspondence between the cable installation components and the electric circuit elements for steady state ampacity calculations. Note that the capacitances play no part in steady state ratings.

To find the ampacity we first note that the potential of every node in the circuit is analog to the temperature of the regions between the layers. Then one substitutes the heat sources by current sources, the thermal resistances by electrical resistances and the thermal capacitances by electrical capacitances. Figure 3 shows the correspondence between the cable installation components and the electric circuit elements for steady state ampacity calculations. Note that the capacitances play no part in steady state ratings.
of the cable's core is the ambient temperature plus \( \Delta t \); see Figure 4.

\[ T = \frac{\rho}{2\pi} \ln \left( \frac{L}{D} \right) \] (7)

Equation (7) is applicable for most internal to the cable layers \( (T_1, T_2, T_3) \). For complicated geometries and for the layers external to the cable, such as three-core cables, duct banks, etc., heuristics are used. For uniformity with (7) the following expression has been proposed:

\[ T = \frac{\rho}{2\pi} G \] (8)

\( G \) is called the geometrical factor because it is a function of the shape and dimensions of the particular geometry under analysis. There are a number heuristics used in the calculation of thermal resistances. For example, there are expressions for: equally or unequally loaded cables, for touching or not touching cables, for flat or triangular formations, trefoils, backfills, duct banks, etc. There are too many possibilities to be considered in this paper, the interested reader can find all the details in list references of this paper. Numerical methods (finite elements) have been used to determine extensions to the geometrical factors when heuristics do not exist.

The external to the cable thermal resistivity is commonly computed assuming that the surface of the earth in the neighborhood of the cable installation is an isothermal. Kennelly made this assumption in 1893 and it is still being used. This assumption allows for the application of the image method to compute the external to the cable thermal resistance \( (T_4) \). The following expression results from the image method:

\[ T = \frac{\rho}{2\pi} \ln \left( \frac{4L}{D} \right) \] (9)

The thermal resistance of the layers external to the cable \( (T_4) \) must also include the duct when present, and the air inside. The duct itself is of tubular geometry and it very easy to model, however, the treatment of the air inside of a duct is a complex matter. The heat transfer is dominated by convection and radiation and not by conduction. There exist simple formulas, which have been obtained experimentally and that work fine for the conditions tested.

**Loss Factors \( \lambda \)**

Loss factors in equation (5) relate to the losses that metallic layers (sheaths, armors, etc.) produce in proportion to the losses of the cable core. These losses include circulating currents and induced currents (eddy currents). The geometrical arrangements are diverse and some are quite complicated.

The bonding used for sheaths (or concentric neutrals) plays a very important role in the current intensity that circulates in them. Thus the losses are very much dependent on the bonding type and the geometrical arrangement of the cables (flat or triangular formation). The possibilities are too many to be discussed in this paper; the interested reader can see all the details in references [1], [3] and [4]. Currently, even finite elements ampacity programs use analytical expressions to compute the losses produced in every layer of the cable installation.
AC Resistance

The operating resistance of a cable is a function of the temperature and the frequency. The temperature variation is described by:

\[ R(t) = R_0[1 + \alpha(t - t_0)] \] (10)

where:

- \( R_0 \) = Resistance at a base temperature (\( t_0 = 20^\circ C \))
- \( \alpha \) = Coefficient of variation with temperature

Although there exists an analytical expression, using Bessel functions, for the modeling of eddy current effects in cables, for low frequencies (50 and 60 Hz), there are very simple and accurate formulas adequate for ampacity calculations. The eddy current effects are included by two factors. One considers the skin effect (\( y_s \)) and the other, the proximity effect (\( y_p \)). The mathematical expression to account for these losses is:

\[ R(f) = R_{oc}(1 + y_s + y_p) \] (11)

Combining (10) and (11) we have:

\[ R_w(t, f) = R_0[1 + \alpha(t - t_0)](1 + y_s + y_p) \] (12)

The values for \( y_s \) and \( y_p \) are computed from simplified analytical expressions particular to each cable core construction (solid, stranded, segmented, etc.).

VI. IEC VERSUS NEHER-MCGRATH

A detailed description of the difference between the two methods can be found in Appendix F of [1]. For steady state ampacity simulations the two approaches are virtually the same. The greatest difference is that the IEC equations use the metric system while Neher-McGrath use the imperial system. Thus equations look very different, but the two methods are equivalent. In the Neher-McGrath method, there are explicit equations for the transient rating, while in the IEC, detailed methodologies are given. In general, IEC methods are more up to date and consider more cases than the Neher-McGrath method. Following is a description of the most important modeling differences:

Eddy Losses

- In the Neher-McGrath approach only the eddy losses for triangular configurations are computed. IEC includes flat formations as well.
- In the IEC standards the magnetic armors are considered, while they are not in the Neher-McGrath method.

Thermal Resistances

- IEC gives expressions for geometric factors of three-core, oil-filled, belted, etc., cables.
- IEC considers more insulation materials than Neher-McGrath.
- IEC makes a distinction between trefoil and flat configurations (touching and not touching) for \( T_4 \).
- IEC considers in detail unequally loaded cables.
- Soil dry-out is considered in IEC.

VII. COMMERCIAL AMPACITY PROGRAMS

The first and most advanced commercial program for cable ampacity calculations is CYMCAP. Its development started in the 1980’s jointly by Ontario Hydro (Hydro One), McMaster University and CYME International, under the auspices of the Canadian Electricity Association (CEATI).

CYMCAP is based on the IEC Standards and features a very friendly GUI (Graphical User Interface). Over 100 companies in close to 50 countries use CYMCAP. This program can compute steady state ampacities and transient ampacities. CYMCAP features a duct bank optimizer and the possibility to handle several duct banks with different thermal resistivities in the same installation.

USAmp is next in the development ladder. It is based on the Neher-McGrath method for steady state ampacity calculations. It supports transients based on the CIGRE report [13]. It has a GUI, but data is entered and displayed mostly in tabular form. USAmp has been used to obtain the IEEE Standard tables published in [2].

ETAP is another tabular program based on the Neher-McGrath method. It does not support transient ampacity calculations. There are other smaller programs such as: PCORP, Underground Cable Ampacity Calculator, etc. with rudimentary GUI’s and calculation engines. Some are royalty free, with no documentation or technical support.

VIII. CYMCAP

CYMCAP is a dedicated computer program for performing ampacity and temperature rise calculations for power cable installations. A description of its main features is given below.

Analytical Capabilities

- Iterative techniques based on the IEC Standards.
- A detailed graphical representation of virtually any type of power cable. This facility can be used to modify existing cables and enrich the program’s cable library with new ones, including single-core, three-core, belted, pipe-type, submarine, sheathed, and armored cables.
- Different cable installation conditions such as directly buried, thermal backfill, underground ducts, duct banks and multiple soil layers with different thermal resistivity.
- Cables in pipes with the pipe directly buried or in a thermal backfill.
- Independent libraries and databases for cables, duct-banks, load curves, heat sources and installations.
- Simulation of cables on riser poles, groups of cables in air, moisture migration, nearby heat sources and heat sinks, etc.
- Different cable types within one installation.
- Non-isothermal earth surface modeling.
- Cyclic loading patterns as per IEC-60853.
- Multiple cables per phase with proper modeling of the sheath mutual inductances, which greatly
influence circulating current losses, and thus de-rating.

- All bonding arrangements for flat and triangular formations are supported with explicit modeling of minor section lengths, unequal cable spacing, etc.

Figure 5 presents a typical graphical display screen of a duct bank installation containing trefoil arrangements, three-core cables and single-phase circuits. Also, any of its cables can be displayed and edited simultaneously.

![Typical CYMCAP Screen](image)

**Figure 5. Typical CYMCAP Screen**

**Transient Analysis**

The program supports transient thermal analyses including the following:

- Ampacity given time and temperature.
- Temperature analysis given time and ampacity.
- Time to reach a given temperature, given the ampacity.
- Ampacity and temperature analysis as a function of time.
- User-defined load profiles per circuit.
- Multiple cables per installation.
- Circuits can be loaded simultaneously or one at a time.

![Typical transient simulation report](image)

**Figure 6. Typical transient simulation report**

Figure 6 shows a graphical display of the results of a transient simulation. In CYMCAP one can display the temperature as a function of time simultaneously with the load curve, the installation arrangement and the cables used.

**Duct Bank Optimizer**

The Duct Bank Optimizer is an add-on module to CYMCAP that allows the user to determine the optimal placement of several circuits within a duct bank. More specifically, the module can recommend the various circuit dispositions within the duct bank in order that:

- The duct bank overall ampacity, i.e. the sum of the ampacities for all circuits, is maximized.
- The duct bank overall ampacity, i.e. the sum of the ampacities for all circuits, is minimized.
- The ampacity of any given circuit is maximized.
- The ampacity of any given circuit is minimized.

![Results of a duct bank optimization simulation](image)

**Figure 7. Results of a duct bank optimization simulation**

Figure 7 presents a 3 by 4 duct bank with three trefoils and one three-phase circuit (one phase per conduit). There are over 110,000 possible combinations. However, CYMCAP has an elaborated mathematical algorithm that prevents the repetitive calculation of equivalent cases, therefore the solution is obtained very efficiently. The left hand side condition in Figure 7 shows the cables placed automatically. On the right-hand side one can see the optimal cable location that maximizes ampacity.

**Multiple Duct Banks**

The Multiple Duct Banks module (MDB) is the extension to CYMCAP designed to determine the steady state ampacity of cables installed in several neighboring duct banks and/or backfills with different thermal resistivity. The module presents a unique solution combining standard and non-standard calculation methods. The module computes the values of $T_4$ (the external to the cable thermal resistance) using finite elements and then the ampacity (or operating temperature) of the cable system is obtained using the IEC standardized solution method. The following capabilities can be highlighted:

- Modeling up to eleven rectangular areas with different thermal resistivity.
- Modeling up to three duct banks in a single installation.
- Modeling one heat source or sink in the installation.
- Computation of the steady state ampacity or temperature.
Figure 8 exemplifies two of the many possibilities that the MDB (Multiple Duck Bank) modeling facilities of CYMCAP can handle.

Validation

CYMCAP has been validated against field tests. In Figure 9 a comparison between time simulations and field tests is presented. One can appreciate that the simulated and measured results match with reasonable accuracy.

IX. SUMMARY

An introduction to the heat transfer mechanisms in underground cable installations was given. An analysis of the possible solution methods of the diffusion equations was presented. A description of the heat sources and thermal resistances of the different layers of a cable installation has been offered. The basic concepts behind the Neher-McGrath method were discussed together with the differences between the IEEE (Neher-McGrath method) and the IEC standards for underground cable installations. A description of the modeling capabilities CYMCAP, CYME’s cable ampacity program, was presented.

X. REFERENCES


XI. BIOGRAPHY

Francisco de León was born in Mexico City in 1959. He received the B.Sc. and the M.Sc. (summa cum laude) degrees in Electrical Engineering from the National Polytechnic Institute (Mexico), in 1983 and 1986 respectively, and obtained his Ph.D. degree from the University of Toronto, Canada, in 1992. He has held several academic positions in Mexico and has worked for the Canadian electric industry. Currently working with CYME International T&D in St. Bruno (Quebec, Canada), he develops professional grade software for power and distribution systems and is the leading technical support of CYMCAP, CYME's cable ampacity program. He has published over a dozen papers in refereed journals (IEEE/IEE), which have been cited over 100 times in journals listed in the Science Citation Index. Francisco is a Senior Member of the IEEE.